GENERALIZED RAINBOW PATTERNS FOR SPHEROIDAL DROPLET CHARACTERIZATION

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Abstract
In this work we present generalized rainbow patterns for spheroidal droplets as computed using a vector ray tracing approach. The structure and shape of these patterns allows the droplet size, refractive index and aspect ratio of the droplets to be inferred. Also results for droplets tilted with respect to the illuminating beam will be discussed.

1 Introduction
Light scattering from a spheroidal droplet in the vicinity of the primary rainbow region has been described by Marston and Trinh [1]. To distinguish such a rainbow from those arising from a spherical droplet, the pattern from spheroidal droplets is called a generalized rainbow pattern. It has been shown that both the equatorial diameter and the relative refractive index can be deduced from the generalized rainbow pattern of spheroidal water droplets [2]. Several salient optical caustics exist in generalized rainbow patterns. Optical caustics have also been observed in the light scattering with white light illumination, in a more complicated caustic’s progression for highly oblate water droplets and inside of liquid droplets [3-8]. Recently, Lock and Xu [9] studied the formation of the rainbow caustic, transverse cusp and hyperbolic umbilic caustics of oblate spheroids by use of Debye series. In past studies the present authors have used a vector ray tracing (VRT) model to study the optical caustic structures near the primary rainbow angle of a spheroidal water droplet; characteristics of the pattern were used to measure the non-sphericity of the droplet [10].

In the present work, the evolution of the optical caustics in terms of the rainbow and hyperbolic umbilic (HU) limiting fringe will be presented. By use of VRT, a relation between rainbow fringe curvature and the droplet shape, in particular the aspect ratio, is established and compared with experimental results. Based on the VRT model and Airy approximation, the quantities size, shape and refractive index can be deduced from the generalized rainbow pattern of a spheroidal droplet. Also the effect of tilt angle of the spheroidal droplet with respect to the illuminating beam will be reported.

2 Results
The optical caustic structure near the primary rainbow scattering angle of a spheroidal water droplet, as simulated using the VRT model, reveals the shape of the rainbow and HU fringes. It can be seen from Fig.1 that the rainbow fringe folds gradually and then unfolds as the aspect ratio increases.

Figure 1 Evolution of the primary rainbow fringe and the HU fringe as the aspect ratio of an oblate water droplet increases.
The cusp caustic first appears on the right side of Fig. 1(b) for droplets with an aspect ratio a/c=1.07. Then the HU fringe unfolds with increasing aspect ratio. A comparison of the curvature of the rainbow fringes found with the VRT simulations to that obtained from experiments is shown in Fig. 2: the curvature has been computed at the apex point of the rainbow fringe.

Figure 2 Comparison of the curvature of the primary rainbow fringe calculated from VRT simulations and that from experimentally observed generalized rainbow patterns.

The curvatures calculated from experiment are shown with error bars corresponding to one standard deviation taken over an ensemble of independent measurements. It can be seen that the agreement between the VRT model and measurements is excellent: most deviations are within the experimental uncertainty. For large aspect ratios (e.g. a/c >1.23), the droplet jitters significantly in the acoustic levitator used to hold it and the generalized rainbow pattern becomes highly instable and blurry. So it is difficult to distinguish the rainbow fringe without using a high-speed camera.

Based on the relation between the rainbow fringe curvature and the aspect ratio of oblate spheroidal droplets (Fig. 2), the aspect ratios ($r_{GRP}$) of spheroidal droplets are calculated from the generalized rainbow pattern and compared to values observed through direct imaging ($r_{Imaging}$), yielding excellent agreement(Fig. 3a). The error bars shown in this figure correspond to one standard deviation computed over many individual measurements. To evaluate the accuracy of computed $r_{GRP}$, the relative error in percentage ($\frac{(r_{GRP} - r_{Imaging})}{r_{Imaging}} \times 100\%$) is given in Fig. 3b. The relative error lies between ±1%, indicating a high achievable accuracy.

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4 References


